CONSERVATION OF LINEAR MOMENTUM

There are phenomena in which interaction between bodies is so fast that it is difficult to measure the forces that are produced between them or the time that the interaction lasts.

For example, how long does the collision between two billiard balls last for? What force does one ball apply on the other? These questions are, no doubt, difficult to answer. Should we give up trying to calculate the result of collisions? Should we leave everything to the billiard player's experience and intuition?

No, physics doesn't give up on trying to explain phenomena that look difficult that easily.

In these cases, the notion of linear momentum and impulse, in addition to the conditions under which linear momentum is conserved, will allow us to make predictions of the speed and direction of the movement after the interaction.

Objectives

· To understand the physical meaning of linear momentum as a magnitude that measures the capacity to act on another in collisions (movements in one dimension).

· To understand the relation between impulse (of a constant force) and linear momentum, in addition to the conservation of linear momentum in the absence of an external impulse.

· To understand the notion of elastic and inelastic collisions.

·To apply the principle of conservation of linear momentum to the calculus of velocities or masses that collide in elastic and inelastic collisions in one dimension.

· To qualitatively understand the changes of direction produced when collisions are not frontal.

· To apply the principle of conservation of linear momentum to the calculus of velocities or masses of particles in the case of the disintegration of a body into two or three fragments.

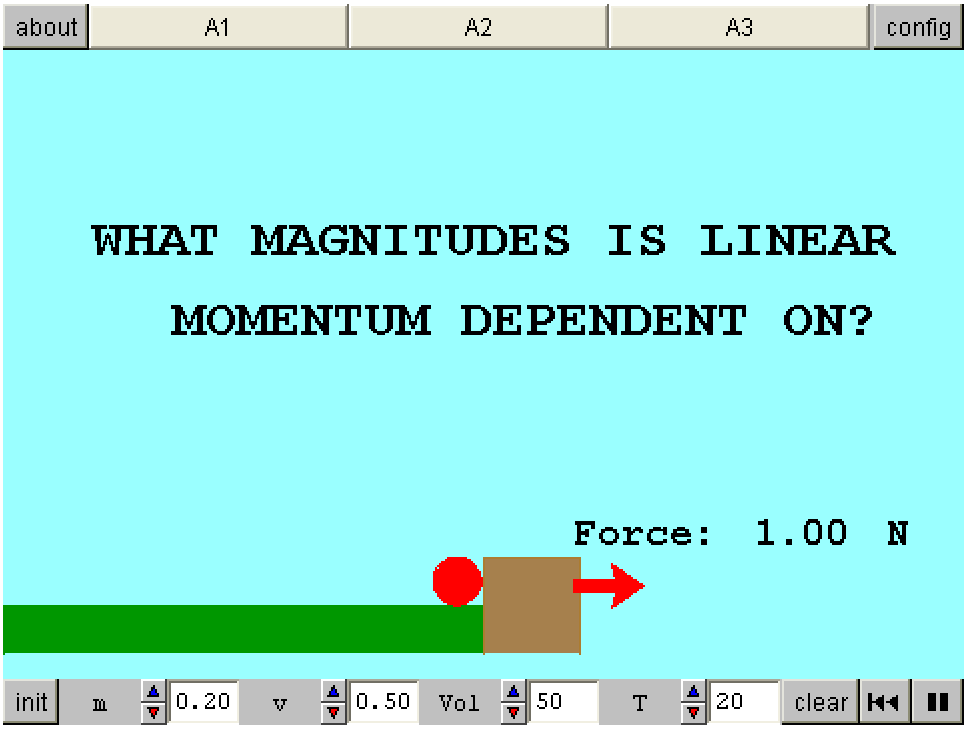
1. What is linear momentum?

You have heard about the notions of position, velocity and acceleration used to describe the movement of a body; you have also heard about using forces. We will now introduce you to another magnitude that is used to relate the bodie's state of motion with the forces that act on it.

We all know that bodies have the capacity of exerting a force on other bodies that are in their way. We will call the magnitude that measures this capacity linear momentum.

We will now try to find out what this magnitude depends on.

In the next visual, a billiard ball collides with the edge of the table, were we have placed an apparatus to measure the maximum force that the ball exerts in the collision. In all cases, we will assume that the collision lasts for a tenth of a second.



A1: You can change the values of the mass, volume, temperature and velocity of the ball before you set it in motion. Make a note in each situation of the force produced when it impacts with the side of the table. According to your experience, what magnitudes is the force produced in the crash dependent on?

A2: How does the force vary if you double or triple the values of each of those magnitudes? Make a note in every case of the values you get altering in each experiment only one of the magnitudes that intervene.
Observe the relation between these magnitudes and the force produced. What do you call this type of relation in mathematics?

A3: Observe all the previous data.

1.2 What is impulse?

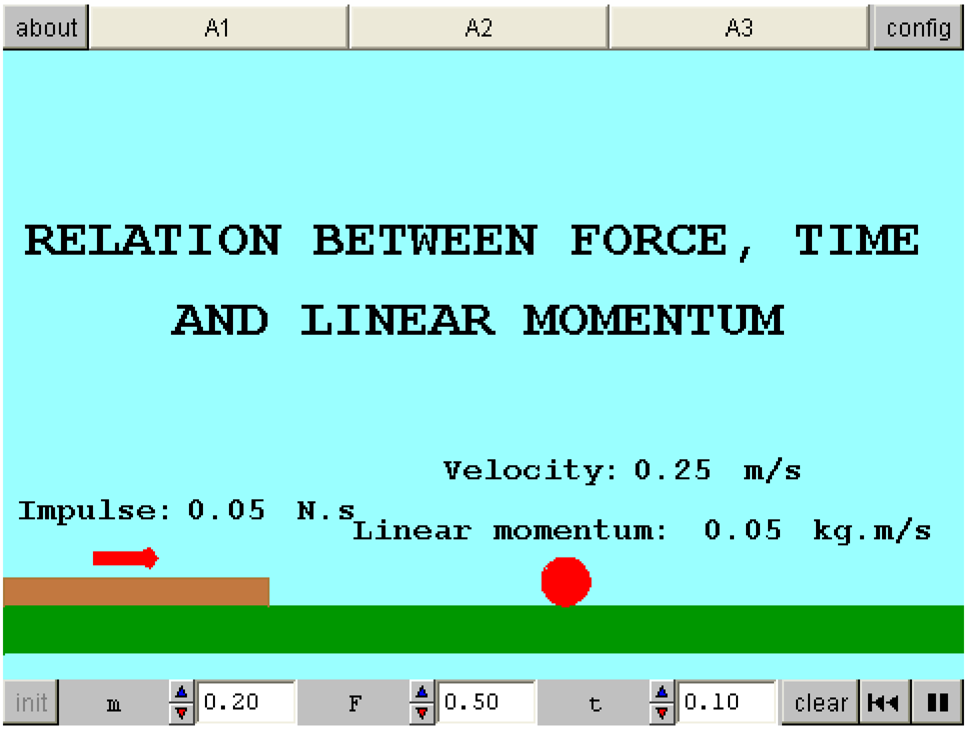
For a billiard ball to have linear momentum, it must have been communicated to it in some way. If we observe someone playing billiards, it is obvious that the linear momentum acquired by the ball depends on the knock it receives from the cue.

We also observe that linear momentum varies after a collision with another ball or with the side of the table.

The magnitude that measures the variation of linear momentum is called impulse.

In the next visual, we will measure the impulse that a billiard ball receives. If you carry out the tasks proposed, you will find out for yourself how to measure the impulse received by a particle.

We use the cue to increase the linear momentum of the billiard ball. The impulse's orientation is the same as the linear momentum's.



A1: Apply different forces on the ball without changing the time. What is the difference between the variation of the linear momentum between the two cases?

A2: Now try changing the time of the impact always maintaining the same force in the experiments. What happens to the linear momentum in each situation?

A3: Now change the mass maintaining the same force and the same time. Do you always get the same final velocity? Do you get the same linear momentum?

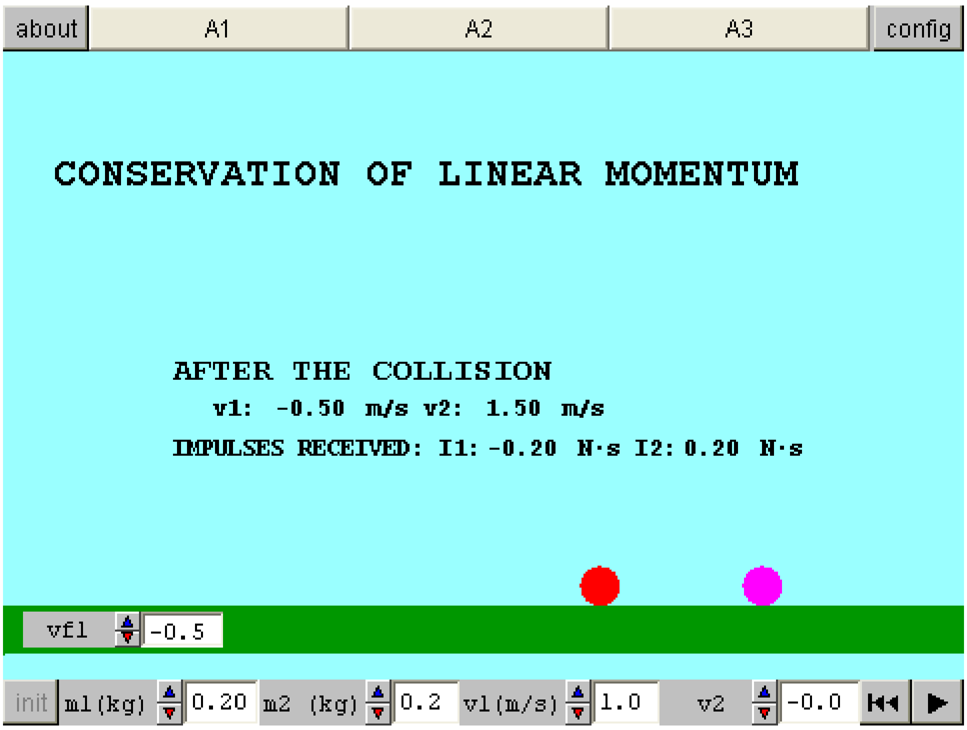
1.3 Conservation of linear momentum

On a billiards table there may be a few balls moving at the same time. **We call the sum of the linear momenta of all of them linear momentum of a particle system**. Observe the fact that, as linear momentum is a vector, when you carry out the sum of linear momenta you must sum them as vectors, not as simple numbers.

When a few balls collide, their individual momenta alter: some decelerate, others accelerate... However, what happens to the combined linear momentum of all the balls?

In the following visual we will try to find out the answer to this question

You can determine the mass and the speed of the balls before the collision and the speed of one of them after the collision.



A1: Set the value of v2 to 0 and perform a few experiments changing the masses of both bodies. Calculate the linear momentum before and after the collision each time (taking the signs into account). What do you observe?

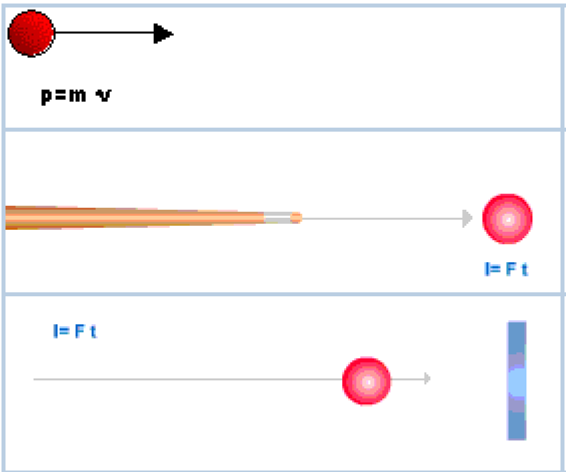
A2: Set the velocities to different values, in such a way that sometimes the balls are moving opposite ways and others they are moving the same way, but one catches up with the other. Each time, calculate the total linear momentum before and after the collision. Do these observations support your observations in the previous exercise?

A3: Observe the fact that you can alter the final velocity of one of the particles. Do it several times without changing the other values.
Observe how the speed of the other particle changes from experiment to experiment although the total linear momentum is conserved. You can even try to produce a collision.

1.4 Conclusions about linear momentum and impulse

We call the magnitude that measures the capacity that a body has to produce an effect on other bodies in a collision **linear momentum**.

We call the variation of linear momentum **impulse**. When we increase the linear momentum of a body, it is receiving a positive impulse; when we diminish its linear momentum, the impulse is negative.



The value of linear momentum is the product: **p=m·v**

Impulse **I=F·t** can increase linear momentum.

It is also possible for impulse **I=F·t** to decrease linear momentum if it is oriented the opposite way.

Principle of conservation of linear momentum: When a particle system does not receive an external impulse, its total linear momentum remains constant.

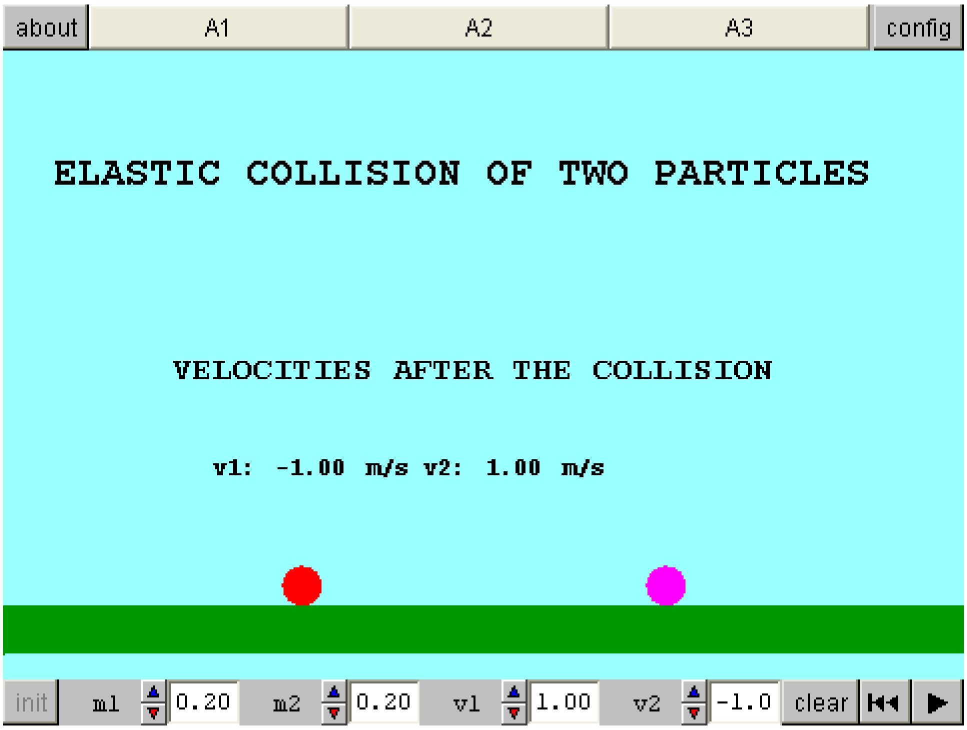
2.1 Elastic collisions

When two bodies collide some of the energy that they carry can be used to deform them or might be dispersed in the form of heat, or maybe the loss of energy is small enough to be neglected.

If the total kinetic energy is conserved in the collision of two particles, then the collision is considered elastic. In this case, conservation of linear momentum and kinetic energy completely determine the velocity of each particle after the collision.

Although we cannot say that there are totally elastic collisions in nature, there are many cases in which the variation of energy in a collision is so small that it cannot be detected. In these circumstances, we will consider the collision elastic.

In the following visual we will observe a frontal collision of two billiard balls that satisfy the conditions of an elastic collision.



A1: By changing the values of one of the balls` mass, write down the velocities acquired.
Are there any cases in which a ball's velocity after the collision is greater than its velocity before the collision?

A2: Leave one of the balls still initially and make both balls have the same mass. What happens after the collision?.
What if the mass of the ball at rest were a lot greater than the other one? In which case is there a greater transference of energy and linear momentum from one ball to the other? (remember that the kinetic energy of a moving body is equal to E=1/2·m·v2)

A3: Try giving one of the balls a small velocity in the same direction as the other ball.
Now verify that the laws of transference of energy and momentum still hold.

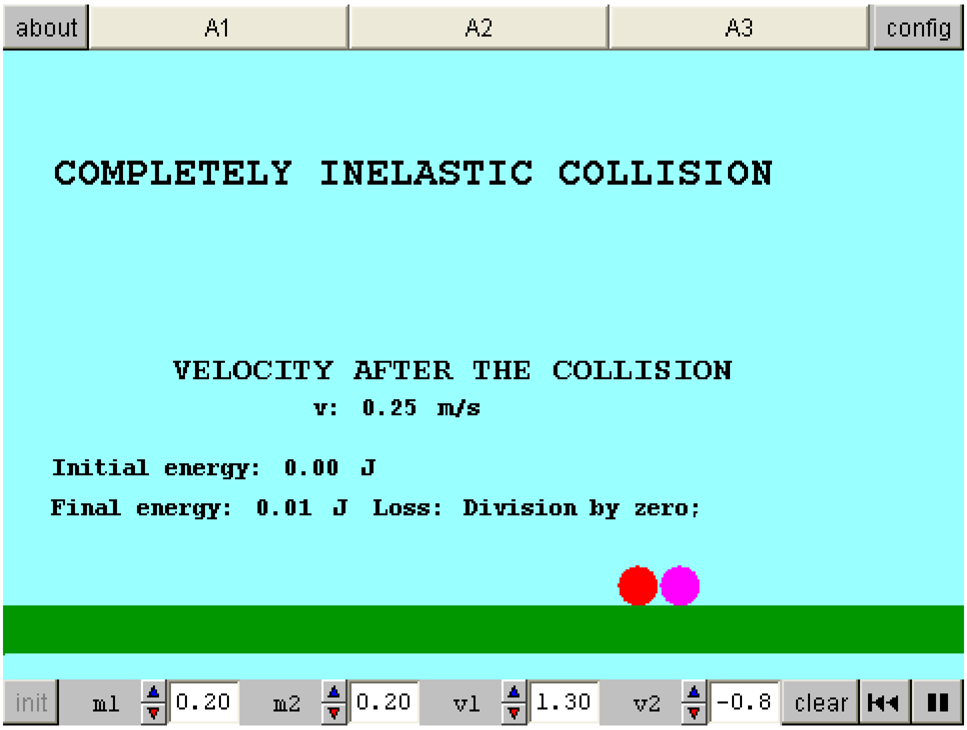
2.2 Completely inelastic collisions

A collision is completely inelastic when the loss of energy is the maximum loss compatible with the conservation of linear momentum. In the case of frontal collisions, this means that both particles end up stuck to one another.

Although there are no cases of completely elastic collisions, there are many cases of completely inelastic collisions.

This is what happens, for example, when a bullet is shot into a block of wood or when an atomic nucleus absorbs a particle in a nuclear reactor.

In the following visual we will assume that a collision of this kind takes place between two billiard balls.



A1: If we vary the mass of one of the two balls, how does the proportion of lost energy vary? Are you able to find a value for the mass that will make both particles stand still after the collision?

A2: Now change the velocity of one of the balls. How does the proportion of lost energy vary? Are you able to find a value of the velocity that will make the particles stand still?

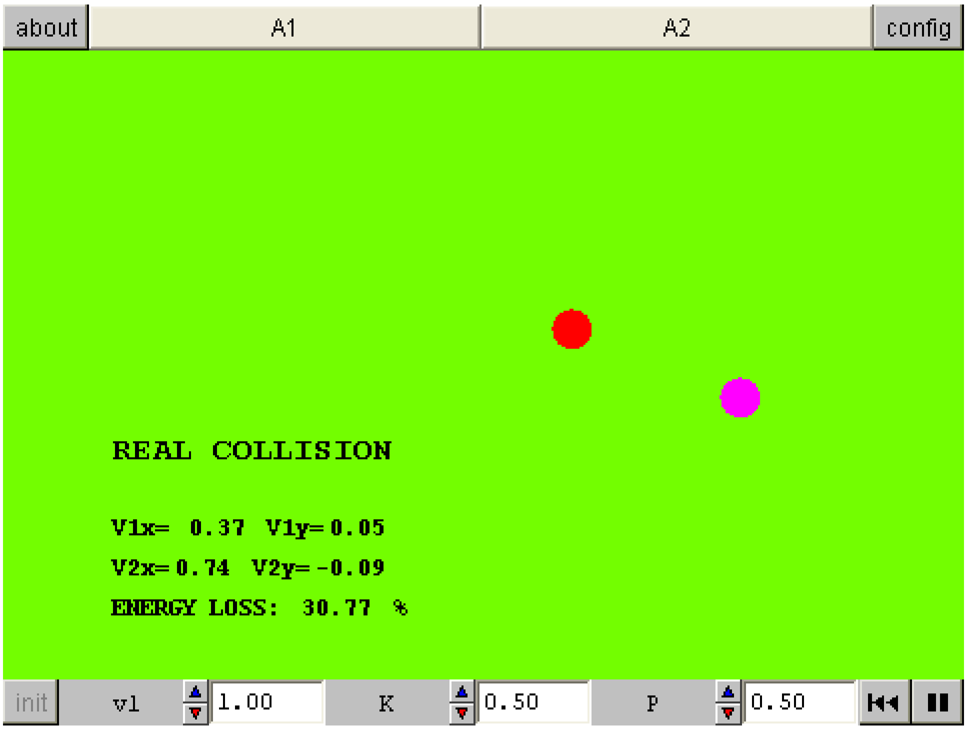
A3: If you have found in the previous exercise the conditions that make the balls stand still, you will be able to answer this question: what condition must their linear momenta satisfy?

2.3 A real collision

The collisions of real particles do not have to be either completely elastic or completely inelastic. There is a coefficient K, known as coefficient of restitution, that can vary from 0 to 1. It measures the degree of elasticity.

In addition, collisions may not be frontal. Therefore, there is an impact parameter P that varies between 1 (for a frontal collision) and 0 (for the case in which one body sweeps past the other).

The change of these two parameters allows the explanation of practically any type of collision between particles. In the following visual we will study the effects of a real collision between two billiard balls. We will assume in every case that one ball is initially at rest while the other is in motion towards it in order to make comprehension easier.



A1: If we make P=1 the collision will be frontal. If you maintain this value giving K values between 0 and 1 you will be able to observe the transition between a completely inelastic collision and a completely elastic collision.

A2: For a set value of K, change the values of P and note down the values of the energy transfered from one ball to another. What is the relation between this transference and P?

2.4 Conclusions drawn from the study of particle collisions

- There are **two ideal cases in which it is possible to determine how a particle will move after a collision:**
- **Frontal elastic collisions**, in which both kinetic energy and linear momenta are conserved.
- **Completely inelastic frontal collisions,** where both particles remain stuck together after the impact. This case produces the maximum loss of energy possible.

- Real collisions oscillate between these two extremes according to the value of the so-called coefficient of restitution. Furthermore, collisions need not be frontal. A tangential collision has an impact parameter value of 0, while in a frontal collision this parameter is equal to 1.

3.1 Particle desintegration into two fragments

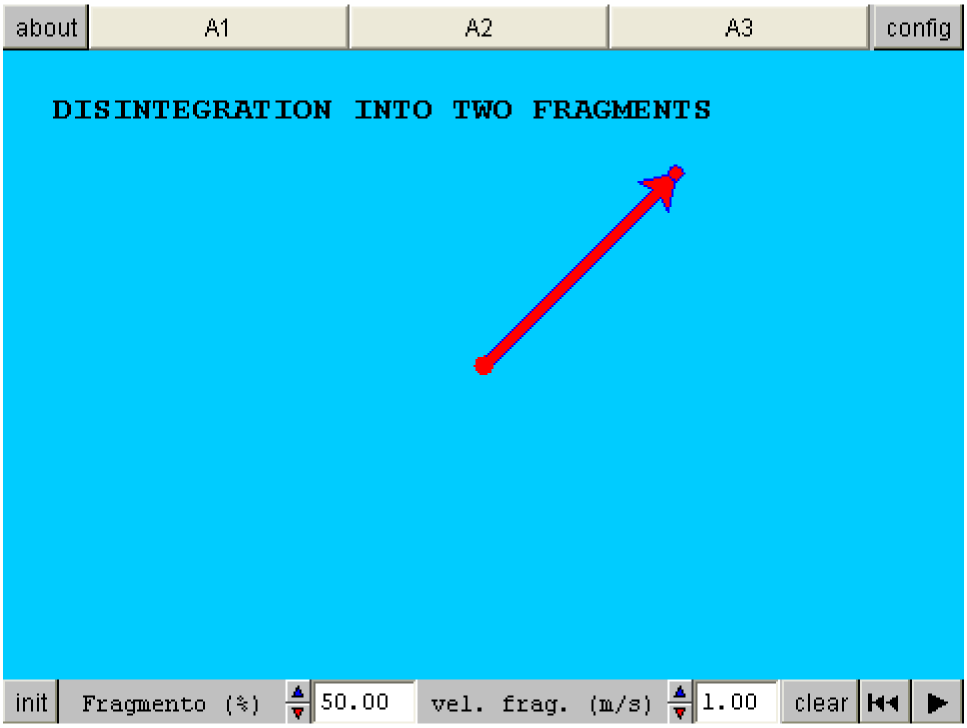
If a body is at rest and it suddenly separates into fragments for internal reasons, then the principle of conservation of linear momentum will still hold because there are still no external forces that modify it.

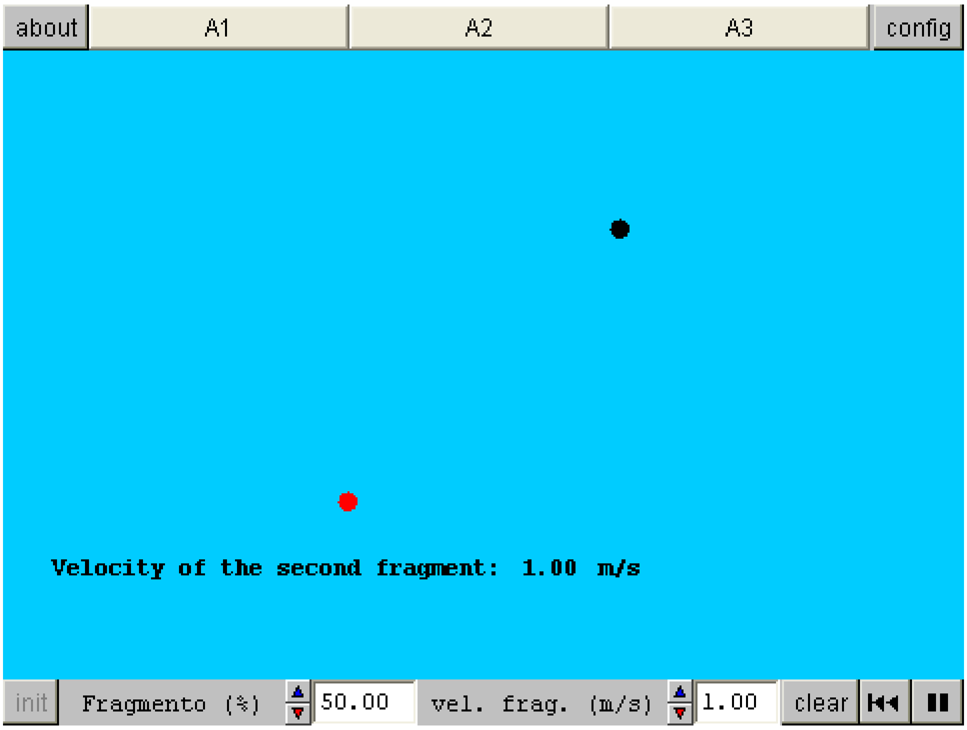
As a consequence of this principle of conservation, if a person standing on ice, for example, throws a snowball in one direction he cannot help moving in the opposite direction. Likewise, if a rifle shoots a projectile, the rifle is pushed the opposite way (unless we hold it tight and resist the backward motion).

In the atomic world, the principle of conservation of linear momentum is also responsible for the following phenomena: if an atomic nucleus disintegrates emitting a particle, the rest of the nucleus must move in the opposite direction.

In the following visual we will see the laws that govern the separation of a body into two fragments.

The mass and velocity of one of the fragments is controlled with numerical data. We can vary its direction dragging the head of the arrow.





A1: Vary the position of the arrow. What can you observe regarding the direction of the fragments in all cases?

A2: Vary the mass of the first fragment between the limits allowed by the program. What does the speed of the second fragment do? Calculate in every case the linear momentum of the two particles. If we take into account that they move in opposite directions, what is the value of the total linear momentum?

A3: A particularly interesting case results when one of the fragments is very light and as fast as possible. This is an approximation of the recoil effect in firearms. Calculate the linear momentum and kinetic energy for both fragments in this case. You will see that, although the bullet and the rifle have equal linear momenta, their kinetic energy is different.

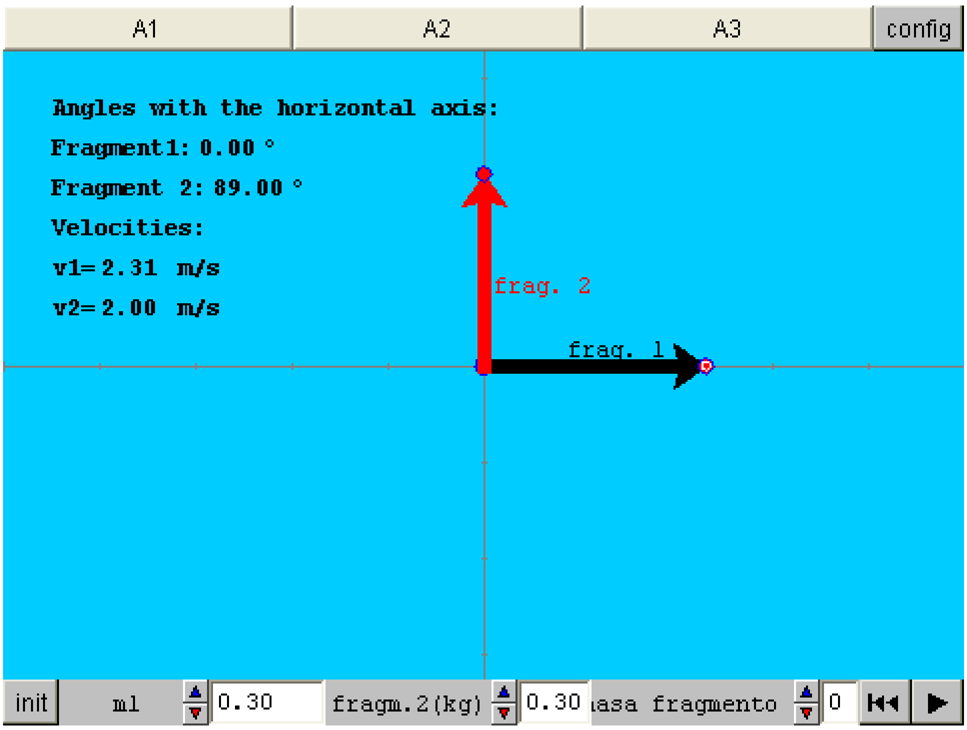
3.2 Particle disintegration into three fragments

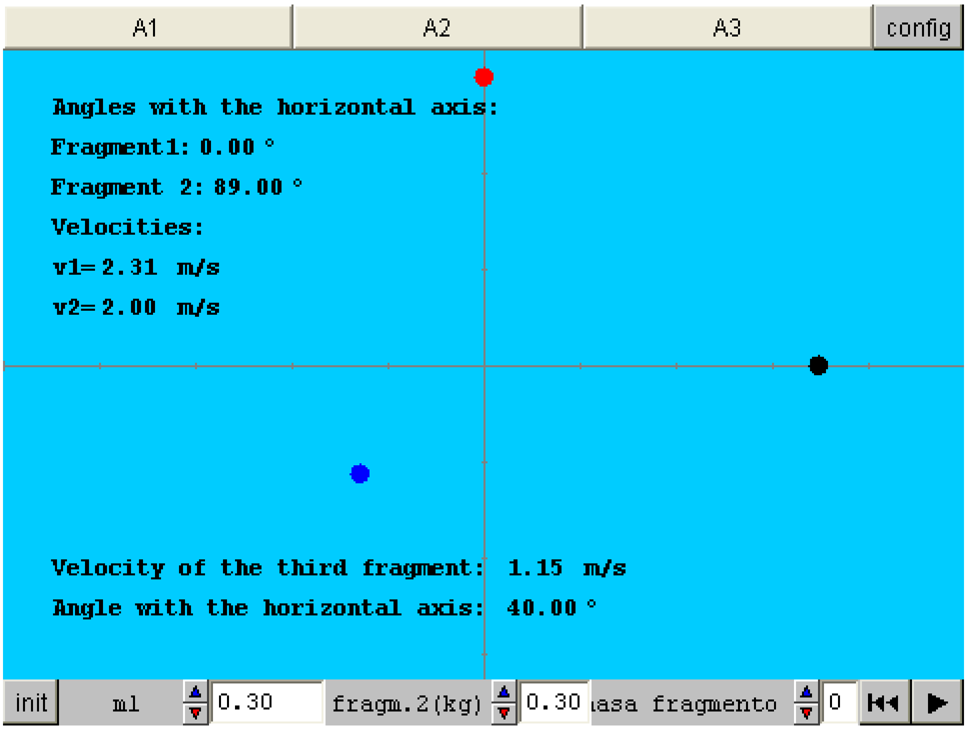
When a body disintegrates, it does not necessarily disintegrate into two fragments; it may disintegrate into two, three or countless fragments.

The explosion of a firework, for example, implies its disintegration into many fragments. Can the fragments have any direction and speed? Well, almost; the conservation of linear momentum limits the possibilities of, at least, one of them.

In the following visual, we see the laws that govern the separation of a body into three fragments, but the conclusions that we will arrive at may be generally valid.

The masses and velocities of the two fragments may be determined numerically. Their direction may be set with the red and black arrows.





A1: Observe the simulation with the initial data. The third fragment (the blue one) must compensate the linear momenta of the other two. Note that its linear momentum and velocity on each axis corresponds to those of the other fragments. You can confirm it altering the velocity (not the direction) of either of the balls or their masses.

A2: If you make the first two fragments leave in the same direction, you will see that the third fragment's linear momentum equals the sum of the other two linear momenta. Observe this fact for a few cases with different masses and velocities.

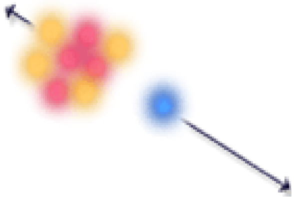
A3: If you make the first two fragments move on the same line in opposite directions, you will verify that the linear momentum for the third fragment equals the difference between the other two. Observe this fact for different cases of masses and velocities.

3.3 Conclusions about particle systems disintegrations

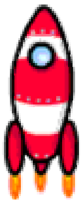
When a system disintegrates into two, three or any number of particles as a consequence of the result of internal forces, the linear momentum of the system must be conserved.

This condition, in every case, makes one of the fragments have a velocity and direction completely determined by the conservation of linear momentum.

Let us look at a couple of examples:



When a radioactive atom disintegrates, the nucleus must move in the opposite direction to the emitted particle, so that linear momentum is conserved.



When a rocket takes off, the mass and velocity of the gases that escape from the rocket in the opposite direction determine the speed that the rocket can achieve due to the conservation of linear momentum.

EVALUATION

Choose the right answers

1 Is it correct to say that linear momentum and impulse are two different names for the same magnitude or are they different magnitudes?

Impulse 1 impulse 2

A They are magnitudes completely independent of one another
B Yes they are synonyms. Therefore, they have the same meaning
C Linear momentum measures the variation of impulse of a body
D Impulse measures the variation of a bodiy`s linear momentum
E They are actually opposite concepts

2 A 0.3 Kg particle with a speed of 1 m/s frontally collides with another particle of the same mass moving in the opposite direction at 2 m/s. The collision is elastic. Calculate the velocity of each of them after the collision.

Elastic collision

A The first one goes at 0.5 m/s and the second one at 1.5 m/s. Both in the same direction as before the collision.
B The first one moves at 0.5 m/s and the second one at 1.5 m/s. Both moving in the direction opposite to the one they were moving in before the collision.
C The first one goes at 2 m/s and the second one at 1 m/s. Both moving in the direction opposite to the one they were moving in before the collision.
D The first one moves at 2m/s and the second one at 1 m/s. Both moving in the same direction as before the collision.

3 After each collision, a particle may change its linear momentum, but not the line along which it was moving.

Real collision

A False, only in frontal collisions does the particle continue to move along the same line.
B False, only in elastic collisions does the particle continue to move along the same line.
C True, if the direction changed, the total linear momentum would not be conserved
D False, only in a completely inelastic collision does a particle continue to move along the same line.

4 Elastic collisions are collisions in which...

elastic collision

A Particles remain stuck together after the collision
B Linear momentum and kinetic energy are conserved
C Linear momentum is conserved, but kinetic energy isn't.
D Kinetic energy is conserved, but linear momentum isn't
E The loss of kinetic energy is about half the total.

5 When no external forces are exerted on a system of particles...

A The kinetic energy of the system is conserved
B The total linear momentum of the system is conserved
C The linear momentum and energy of the system are conserved
D The linear momentum of each particle is conserved
E The mass and velocity of each particle are conserved

6 When a system disintegrates into fragments...

A The total linear momentum and total kinetic energy are conserved.
B The total kinetic energy of the system is conserved
C The linear momentum of each particle is conserved
D The total linear momentum of the system is conserved.

7 The linear momentum of a particle depends on

linear momentum

A The force that the particle carries with it
B The mass, acceleration and temperature of the particle.
C A particle's mass, velocity and density.
D The particle's mass and its velocity

8 Two particles, A and B, with masses of 0.2 and 0.3 Kg respectively, are subjected to a force of 2 N for 3 s.Which of them gains a higher velocity? Which of them gains more linear momentum?

Momentum and impulse

A The heavier one. This way they both gain the same amount of linear momentum.
B Both of them gain the same velocity, because they have received the same impulse
C The lighter one. This way both particles will gain the same amount of linear momentum.

9 A particle at rest disintegrates into two fragments. The first fragment (20% of the total mass) gains a velocity of 2 m/s. What velocity will the other fragment gain?

Disintegration

A 3 m/s in the opposite direction
B 1 m/s in the opposite direction
C 0,75 m/s in the same direction
D 0,75 m/s in the opposite direction
E 3 m/s in the opposite direction

10 Completely inelastic collisions are collisions in which...

comp. inelastic collisions

A Linear momentum is conserved, but kinetic energy isn’t.
B Kinetic energy is conserved, but linear momentum isn’t
C About half the kinetic energy is lost
D Linear momentum and kinetic energy are conserved
E Particles remain stuck together after the collision